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MEng Individual Project

Lemmagen: Extending the WebAssembly Specification Toolchain for Theorem Provers

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June 25, 2025

Background

WebAssembly

WebAssembly (Wasm) is a low-level bytecode language [HRS+17].

- Runs on the Web for client and server applications.
- Compilation target for major programming languages.
- Rapidly evolving with rising industry adoption.



WebAssembly

Wasm is characterised by its strong formalism [Ros25].

if $blocktype instr_1^*$ else $instr_2^*$ end

- The block type must be valid as some function type $[t_1^*] \to [t_2^*]$.
- Let C' be the same context as C, but with the result type $[t_2^*]$ prepended to the labels vector.
- Under context C', the instruction sequence $instr_1^*$ must be valid with type $[t_1^*] \to [t_2^*]$.
- Under context C', the instruction sequence $instr_2^*$ must be valid with type $[t_1^*] \to [t_2^*]$.
- Then the compound instruction is valid with type $[t_1^* \ \text{i32}] \to [t_2^*]$.

```
\frac{C \vdash blocktype: [t_1^*] \rightarrow [t_2^*] \qquad C, \mathsf{labels}\,[t_2^*] \vdash instr_1^*: [t_1^*] \rightarrow [t_2^*] \qquad C, \mathsf{labels}\,[t_2^*] \vdash instr_2^*: [t_1^*] \rightarrow [t_2^*]}{C \vdash \mathsf{if}\,\,blocktype\,\,instr_1^*\,\,\mathsf{else}\,\,instr_2^*\,\,\mathsf{end}: [t_1^*\,\,\mathsf{i32}] \rightarrow [t_2^*]}
```

WebAssembly

Wasm even includes **soundness theorems** in its specification [Wor22a]:

Theorem (Progress). If a configuration S; T is valid (i.e., $\vdash S; T : [t^*]$ for some result type $[t^*]$), then either it is terminal, or it can step to some configuration S'; T' (i.e., $S; T \hookrightarrow S'; T'$).

Theorem (Preservation). If a configuration S;T is valid with result type $[t^*]$ (i.e., $\vdash S;T:[t^*]$), and steps to S';T' (i.e., $S;T\hookrightarrow S';T'$), then S';T' is a valid configuration with the same result type (i.e., $\vdash S';T':[t^*]$). Furthermore, S' is an extension of S (i.e., $\vdash S \leq S'$).

WasmCert

This formalism enables a precise mechanisation of Wasm.

WasmCert-Rocg and WasmCert-Isabelle provide a complete mechanisation of Wasm in Rocq and Isabelle/HOL [WRPP+21].

```
Inductive be_typing : t_context -> seq basic_instruction -> function_type -> Prop :=
| bet const : forall C v, be typing C [::BI const v] (Tf [::] [::typeof v])
bet unop : forall C t op.
   unop_type_agree t op -> be_typing C [::BI_unop t op] (Tf [::t] [::t])
bet binop : forall C t op.
   binop type agree t op -> be typing C [::BI binop t op] (Tf [::t; t] [::t])
```

Problem

Formal rules need to be translated into multiple formats.

- Declarative representations in LATEX format
- Algorithmic representations in prose format
- Reference interpreter in OCaml
- Test suite in .wast format
- Mechanised definitions in Coq

Manual translation is just too tedious!

- Prone to human error.
- Not scalable as the Wasm specification continues to grow.

SpecTec

SpecTec DSL serves as a "single source of truth" from which key artefacts can be auto-generated [YSL+24].

Not limited to Wasm (but some backends are specialised).

Wasm 3.0 specification will be produced with SpecTec [Ros25]!

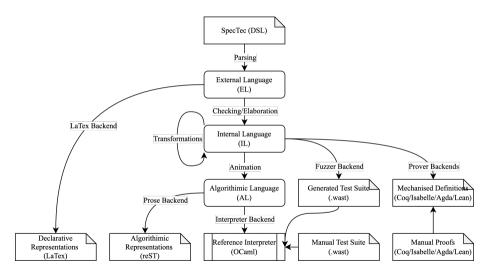


SpecTec has been adopted

Published on March 27, 2025 by Andreas Rossberg .

Two weeks ago, the Wasm Community Group voted to adopt SpecTec for authoring future editions of the Wasm spec. In this post, I'll shed some light on what SpecTec is, what it helps with, and why it takes Wasm to a new level of rigor and assurance that is unprecedented when it comes to language standards.

SpecTec



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IL2Rocq

IL2Rocq is the work of Diego, a master's student from last year [Cup24].

Major achievements by Diego:

- Implemented a SpecTec backend for Rocq mechanised definitions
- Completed the preservation proof using auto-translated Rocq definitions!

1. **2017~**: Wasm 1.0 draft is released [HRS+17]

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- 5. **2024**: Preservation proof is completed [Cup24]
- 6. **2025**: Progress proof is completed (NEW)
- 7. 2025: Lemmagen is developed (NEW)

Project

Contributions

- Progress Proof
- Decidable Equality Proofs
- First-Order Logic Extension in DSL
- Template Mechanism in DSL

Project

Progress Proof

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Progress Proof

Continuation of Diego's work on IL2Cog and the preservation proof [Cup24].

- Proof by mutual induction on the structures of 2-3 typing relations.
- Accommodates key differences between the SpecTec DSL and WasmCert-Rocg.

```
Theorem t_progress :
 forall s f es ts.
  Config ok (config (state s f) es) ts ->
  terminal form es \/
  exists s' f' es'.
  Step (config__ (state__ s f) es) (config__ (state__ s' f') es').
```

Progress Proof

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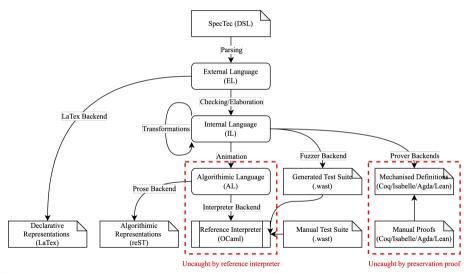
```
Lemma t_progress_e: forall s C C' f vcs es tf ts1 ts2 lab ret,
 Admin instrs ok s C es tf ->
 tf = functype_ ts1 ts2 ->
 C = (upd_local_label_return C' (map typeof f.(frame__LOCALS)) lab ret) ->
 Module instance ok s f.(frame MODULE) C' ->
 map typeof vcs = ts1 ->
 Store_ok s ->
 not lf br es ->
 not_lf_return es ->
 terminal_form (list__val__admininstr vcs ++ es) \/
 exists s' f' es', Step (config (state s f) (list val admininstr vcs ++ es))
                        (config (state s' f') es').
```

Modifications to DSL

Key errors in the DSL found during the proof development (mostly acknowledged by Andreas and Conrad):

```
+rule Step/ctxt-seq:
+ z; val* admininstr* admininstr''* ~> z'; val* admininstr'* admininstr''*
+ -- Step: z; admininstr* ~> z'; admininstr'*
rule Step/ctxt-frame:
- s; f; (FRAME_ n `{f'} admininstr*) ~> s'; f; (FRAME_ n `{f'} admininstr'*)
- -- Step: s; f'; admininstr* ~> s'; f'; admininstr'*
+ s: f: (FRAME n `{f'} admininstr*) ~> s': f: (FRAME n `{f''} admininstr'*)
+ -- Step: s: f': admininstr* ~> s': f'': admininstr'*
```

Modifications to DSL



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Project

Decidable Equality Proofs

Contributions

- Progress Proof
- Decidable Equality Proofs
- First-Order Logic Extension in DSL
- Template Mechanism in DSL

Motivation

Decidable equality is essential for case analysis in Rocq proofs.

- In **Rocq proofs**, we must show $x = y \lor x \neq y$ explicitly (constructive logic).
- In **hand-written proofs**, showing $x = y \lor x \ne y$ is not necessary (classical logic).

Manually proven for each data type in the progress proof.

Automated by extending IL2Rocq.

Decidable Equality Proofs

Based on WasmCert-Rocq's approach [BGP+25].

- Fully automated proofs for all data types.
- Proofs are significantly simplified compared to WasmCert-Rocq (about 100 to 10 lines).

```
Ltac rect' build projection proj rect :=
  let t :=
    lazymatch type of rect with
    | forall P : ?t -> Type, => t
    end in
  let g := rect' type projection proj rect in
  refine (_ : g);
Definition administrative_instruction_eq_dec : forall e1 e2 :

→ administrative instruction.

  \{e1 = e2\} + \{e1 <> e2\}.
Proof. decidable_equality_using

→ administrative instruction rect'. Defined.
```

Decidable Equality Proofs

Based on WasmCert-Rocq's approach [BGP+25].

- Fully automated proofs for all data types.
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```
Create HintDb eq_dec_db.
Ltac decidable_equality_step :=
   do [ by eauto with eq_dec_db | decide equality ].
Fixpoint administr_eq_dec (v1 v2 : administr ) {struct v1} :
   {v1 = v2} + {v1 <> v2}.
Proof. decide equality; do ? decidable_equality_step. Defined.
```

Project

First-Order Logic Extension in DSL

Contributions

- Progress Proof
- Decidable Equality Proofs
- First-Order Logic Extension in DSL
- Template Mechanism in DSL

Motivation

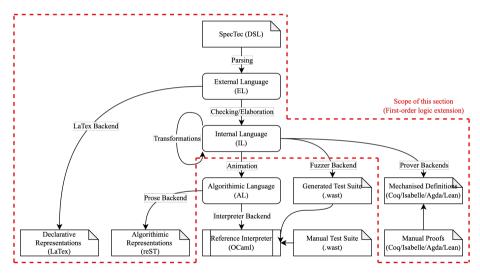
More theorems are expected in the Wasm 3.0 draft [Wor22b]:

- Principal types
- Type lattice
- Properties on compositionality of instruction sequences

Makes sense to specify theorems in the DSL as a "single source of truth" [YSL+24]:

- Enforces uniform style of theorem statements across theorem provers.
- Allows translation of theorems into LaTEX and prose formats.
- Guarantees that theorems in the specification precisely match the mechanised definitions.

Scope



Design

DSL syntax extensions and examples:

```
def ::=
                                          exp ::=
                                            "@" "(" relid ":" exp ")"
  "theorem" thmid ": " exp hint*
  "lemma" thmid ":" exp hint*
                                            "@" "(" exp ")" iter*
  "theorem" thmid hint* "=" exp
                                            forall args exp
  "lemma" thmid hint* "=" exp
                                            exists args exp
              theorem t_progress =
                forall (s, f, admininstr*, t?)
                @(Config ok: |- s: f: admininstr* : t?) =>
                $terminal form(admininstr*) \/
                exists (s', f', admininstr'*)
                @(Step: s; f; admininstr* ~> s'; f'; admininstr'*)
```

Results

Produced by Coq, LATEX and prose backends:

Theorem (Progress)

For all s, f, $instr^*$, $t^?$, if $instr^*$ is valid with type $t^?$ under the context f and the store s, then $instr^*$ is in terminal form or there exists s', f', $instr'^*$ such that $((s, f), instr^*)$ steps to $((s', f'), instr'^*)$

Project

Template Mechanism in DSL

Contributions

- Progress Proof
- Decidable Equality Proofs
- First-Order Logic Extension in DSL
- Template Mechanism in DSL

Motivation

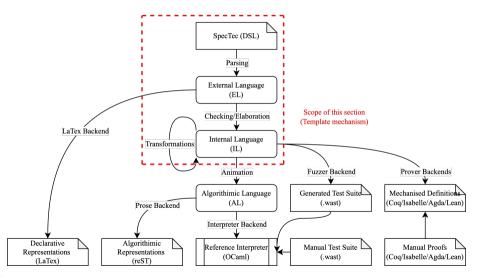
Proofs tend to be lengthy (about 2000-3000 lines).

Long proofs should be extracted into **auxiliary lemmas**. e.g. Individual cases, induction steps and inversion lemmas.

- Enhances readability and maintainability.
- Enforces finer details of proof structure across theorem provers.

But the statements of these lemmas can be quite repetitive!

Scope



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Design

Auxiliary lemmas without template mechanism:

```
lemma Step_pure__preserves_nop = forall (s, C, ft)
  @(Admin_instrs_ok: s; C |- NOP : ft) =>
  @(Step-pure: NOP ~> eps) =>
  @(Admin_instrs_ok: s; C |- eps : ft)

lemma Step_pure__preserves_local_tee = forall (s, C, ft, val, x)
  @(Admin_instrs_ok: s; C |- val (LOCAL.TEE x) : ft) =>
  @(Step-pure: val (LOCAL.TEE x) ~> val val (LOCAL.SET x)) =>
  @(Admin_instrs_ok: s; C |- val val (LOCAL.SET x) : ft)
```

Design

Auxiliary lemmas with template mechanism:

Results

Produced by Cog backend:

```
Lemma Step pure preserves nop : forall (v ft : functype) (v C : context) (v s :

→ store), ((Admin instrs_ok v_s v_C [(admininstr_NOP)] v_ft) -> ((Step pure))
→ [(admininstr NOP)] []) -> (Admin instrs ok v s v C [] v ft))).
Proof Admitted
Lemma Step_pure__preserves__local_tee : forall (v_x : idx) (v_val : val) (v_ft :
   functype) (v C : context) (v s : store), ((Admin_instrs ok v_s v_C [(v val :
   admininstr):(admininstr LOCAL TEE v x)] v ft) -> ((Step pure [(v val :
   admininstr):(admininstr LOCAL TEE v x)] [(v val : admininstr):(v val :
   admininstr); (admininstr LOCAL SET v x)]) -> (Admin instrs ok v s v C [(v val

→ : admininstr); (v val : admininstr); (admininstr LOCAL SET v x)] v ft))).
Proof Admitted
```

Evaluation

Progress Proof

Key strengths:

Identified **key errors in the DSL** [YSL⁺25].

- Not detected by the reference interpreter nor the preservation proof in SpecTec!
- Highlights the importance of developing both proofs in this project.

Identified key issues in IL2Rocq [Cup24].

- Handling of conjunction of premises
- Handling of iteration dimensions
- Translation of list slicing notations

Some lemmas were reformulated from scratch (not just a direct porting process).

Progress Proof

Consistent use of **SSReflect syntaxes**.

- Enhances readability and maintainability.
- WasmCert-Rocq contains a mixture of both Rocq and SSReflect syntaxes.

```
Goal 1
l, c : memaddr
ts1, ts2 : seg valtype
C : context
s : store
Hstep : Step pure
          [:: admininstr_CONST (valtype_INN inn_I32) (c : val_);
              admininstr BR TF 11 [:: admininstr BR 1]
Hval: (c : val ) ≠ 0
ts1_comp, ts2_comp, ts3_comp, ts4_comp : seq valtype
H1 comp : ts1 = ts1 comp ++ ts2 comp
H2 comp : ts2 = ts1 comp ++ ts3 comp
ts1 comp0, ts2 comp0, ts3 comp0, ts4 comp0 : seg valtype
H1 comp0 : ts2 comp = (ts1 comp0 ++ ts2 comp0)%list
H2 comp0 : ts4 comp = (ts1 comp0 ++ ts3 comp0)%list
H3 comp0 : ts2 comp0 = ts4 comp0
H4 comp0 : Admin instr ok s C
             (admininstr CONST (valtype INN inn I32) c)
             (functione ts4 comp0 ts3 comp0)
ts1 comp1, ts2 comp1, ts3 comp1, ts4 comp1 : seg valtype
H1 comp1 : ts4 comp = (ts1 comp1 ++ ts2 comp1)%list
H2 comp1 : ts3 comp = (ts1 comp1 ++ ts3 comp1)%list
H3 comp : ts2 comp1 = ts4 comp1
ts : seg valtype
ts': resulttype
H1 : ts3 comp1 = ts ++ ts'
H2: ts4 comp1 = ts3 comp1 ++ [:: valtype INN inn I32]
H3 : (1 < Datatypes.length (context LABELS C))%cog nat
H4 : lookup total (context LABELS C) l = ts'
Admin instrs ok s C [:: admininstr BR l] (functype ts1 ts2)
```

```
Goal 1
1. c : memaddr
C : context
s : store
Hstep: Step pure
          [:: admininstr CONST (valtype INN inn I32) (c : val ):
              admininstr BR IF 1] [:: admininstr BR 1]
Hval : (c : val ) # 0
ts1, ts2, ts, ts1', ts2', ts3, ts', ts1'', ts3' : seg valtype
Hoonst: Admin instrok s C (admininstr CONST (valtype INN inn I32) c)
          (functype__ ts1'' ts3')
ts'', ts3'', ts2''; seg valtype
Hts' : ts' ++ ts3' = ts'' ++ ts3''
ts''' : seg valtype
tslab : resulttype
Hts2'' : ts2'' = ts''' ++ tslab
Hts3'' : ts3'' = ts2'' ++ [:: valtype INN inn I32]
Hlookup : lookup total (context LABELS C) l = tslab
Hlen : 1 < Datatypes, length (context LABELS C)
Admin instrs ok s C [:: admininstr BR l]
(functype (ts ++ ts' ++ ts1'') (ts ++ ts'' ++ ts2''))
```

Progress Proof

Future work:

Pending tasks for Wasm mechanisation:

- Update the preservation proof to accommodate the changes in the DSL/IL2Coq.
- Upgrade the proofs to Wasm 2.0 [Wor22a].
- Mechanisation of numerics and module instantiation [Wor22a].

First-Order Logic Extension in DSL

Key strengths:

- Backwards compatible/minimally intrusive design.
- Correctness confirmed through visual comparison and migration of proofs.

Room for improvements:

- Support quantification of variables of family types.
- Support updating theorem statements iteratively.
- The @ symbol is currently required for disambiguation in the LR(1) grammar.

Template Mechanism in DSL

Key strengths:

- Backwards compatible/minimally intrusive design.
- Correctness confirmed through visual comparison and migration of proofs.
- Designed as a generic mechanism.
- Allows quantification of free variables in the substituted expressions.

Room for improvements:

- Handle name conflicts with free variables in the substituted expressions.
- Formalise the process of template expansion.

Conclusion

Contributions

- Progress Proof
- Decidable Equality Proofs
- First-Order Logic Extension in DSL
- Template Mechanism in DSL

Contributions

These are all interconnected (not random)!

- Progress Proof
 - ↓ Identified areas of improvement in SpecTec/IL2Rocq
- Decidable Equality Proofs
 - ↓ Required as part of Cog translation
- First-Order Logic Extension in DSL
 - Required to specify template definitions
- Template Mechanism in DSL

Conclusion

Challenges overcome in this project:

- Lots of background material (e.g. Rocq, SSReflect).
- Lots of code reading (e.g. WasmCert-Rocq, SpecTec, IL2Coq).
- Lots of proofs and coding (e.g. frontend, middlends, backends in SpecTec).
- Exploring various design choices for Lemmagen.

Lemmagen only serves as a proof of concept at the moment.

But the reactions have been positive so far!

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Thank You

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SpecTec Highlight

For those having a hard time reading the DSL

```
;; Memory instructions
rule Step_read/load-num-trap:
 z: (CONST (INN I32) i) (LOAD t mo) ~> TRAP
 -- if (i + mo.0FFSET + $size(t)/8 > |\$mem(z, 0).BYTES|)
rule Step_read/load-num-val:
 z; (CONST (INN I32) i) (LOAD t mo) ~> (CONST t c)
 -- if bytes(t, c) = mem(z, 0).BYTES[i + mo.0FFSET : <math>size(t)/8]
```

Errata I

Minor corrections made to the report since its submission:

Location	Error	Correction
p.32-33	Inappropriate use of repeat tactical in Fig-	Replaced with do ? tactical to match
	ures 3.26 and 3.27	SSReflect-native style
p.30, par 5, line 2	Inaccurate description of the proof in Figure	Clarified that the proof proceeds by estab-
	3.24	lishing the Config_sound co-recursively
p.65, par 3, line 2	Unclear discussion of the limitation imposed	Clarified that this limitation is particularly rel-
	on iterative proof development	evant due to frequent updates
p.21, par.3	Missing description of the modifications to	Clarified that this rule has been modified to
	the Step/ctxt-frame rule	allow changes in the frame state f

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